

Testing Two Independent Means

Chapter 31 described a t test for comparing a sample mean, a statistic, which is a proxy for a population mean, a parameter, to a hypothesized population mean. This test is known as a one-sample t test. This chapter describes a t test for comparing the means of two independent samples, which are proxies for two population means. This test is known as a two-sample t test. To start, review the methodologies described in Chapters 23 and 30.

Sample Data

SAT scores are approximately normally distributed. Table 32.1 shows a random sample of the SAT scores of 25 male students and the SAT scores of 25 female students.

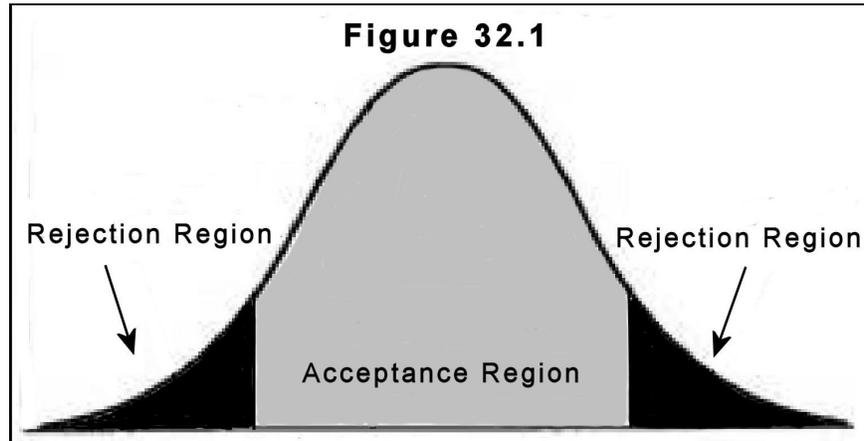
Table 32.1				
SAT Scores - Males				
504	519	462	444	530
347	538	561	709	463
640	427	450	538	469
410	521	520	494	392
369	431	487	361	506
SAT Scores - Females				
357	573	422	544	619
619	478	563	416	653
438	414	721	620	560
665	394	504	434	478
474	348	586	379	581

Null Hypothesis and Alternative Hypothesis

Say that one states that the average SAT scores of male students is equal to the average SAT scores of female students. That is, $H_0: \mu_m = \mu_f$ and $H_1: \mu_m \neq \mu_f$. Is it true?

Figure 32.1 on Page 130 shows that, if the the test shows that the calculated t score is within the acceptance region, shown in gray, then, one fails to reject—that is, accepts—the null hypothesis and, therefore, is likely that there is no difference between the average SAT scores of the male students and the average SAT scores of the female

students. Conversely, if the test shows that the the calculated t score is within the rejection regions, shown in black, then, one rejects the null hypothesis and, therefore, is likely that there is a difference between the average SAT scores of the male students and the average SAT scores of the female students.



Sample Means

The sample mean of the two distributions shown in Table 32.1 on Page 129 are 483.64 and 517.60:

$$(504+347+\dots+506)/25=483.64$$

$$(357+619+\dots+581)/25=517.60$$

Sample Standard Deviations

The sample standard deviation of the two distributions shown in Table 32.1 on Page 129 are 82.86 and 104.06:

$$\text{SQRT}(((504-483.64)^2+(347-483.64)^2+\dots+(506-483.64)^2)/24)=82.86$$

$$\text{SQRT}(((357-517.60)^2+(619-517.60)^2+\dots+(581-517.60)^2)/24)=104.06$$

T Score

The formula for calculating the t score is ...

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

... where t is the t score, x-bar are the sample means, μ are the population means, s are the sample standard deviations, and n are the sample sizes.

For this example, (μ₁-μ₂)=0 and the relevant t score is -1.277:

$$(483.64 - 517.60 - 0) / \text{SQRT}((82.86^2/25) + (104.06^2/25)) = -1.277$$

Degrees of Freedom

The formula for calculating the degrees of freedom is ...

$$DF = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

... where DF is the degrees of freedom, s are the standard deviations, and n are the sample sizes.

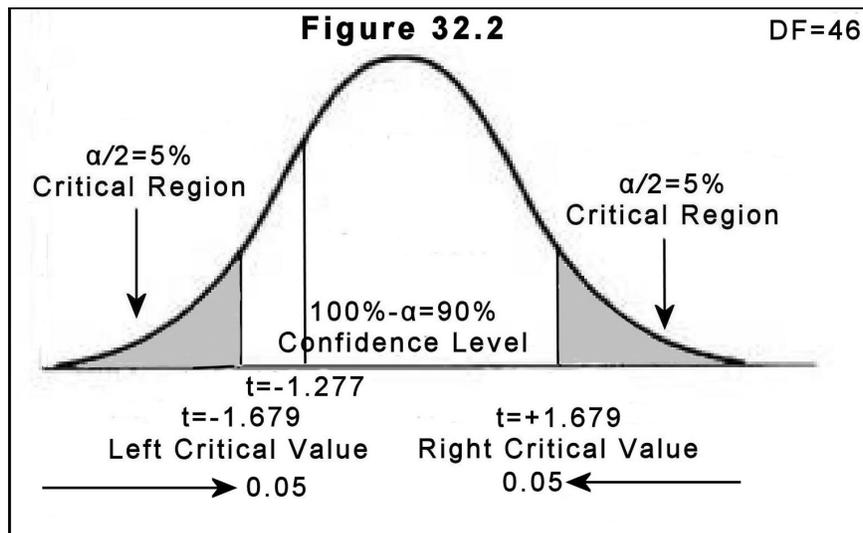
For this example, DF=46 (rounded):

$$((82.86^2/25 + 104.06^2/25)^2) / (((82.86^2/25)^2 / (25 - 1)) + ((104.06^2/25)^2 / (25 - 1))) = 45.71$$

Confidence Level and Critical Values

The t table is used for finding the critical values (CVs) of t that correspond to a selected level of confidence, where CL=100%-α, which, for this example, 90% is selected. Table 32.2 on Page 132 and Figure 32.2 on Page 132 show, for DF=46, that the t scores that correspond to the 90% confidence level, which is CL=100%-10%, are -1.679 for the left side and +1.679 for the right side. The confidence level shows the probability, which is 90%, that the t scores are between -1.679 and +1.679. Note that α/2=5.0%.

Table 32.2			
Degrees of Freedom	Probability (One Tail)		
46	0.1000	0.0500	0.0250
	1.300	1.679	2.013
	Probability (Two Tail)		
	0.1000 (0.0500+0.0500)	0.0500 (0.0250+0.0250)	0.0250 (0.0125+0.0125)
	1.679	2.013	2.317



Inference

Figure 32.2 shows that, for DF=46, the t score of -1.277 is between the critical values of -1.679 and +1.679 and within the 90% confidence level. Therefore, one does not reject the null hypothesis. That is, one can say that, most likely, there is no difference between the average SAT scores of male students and the average SAT scores of female students.

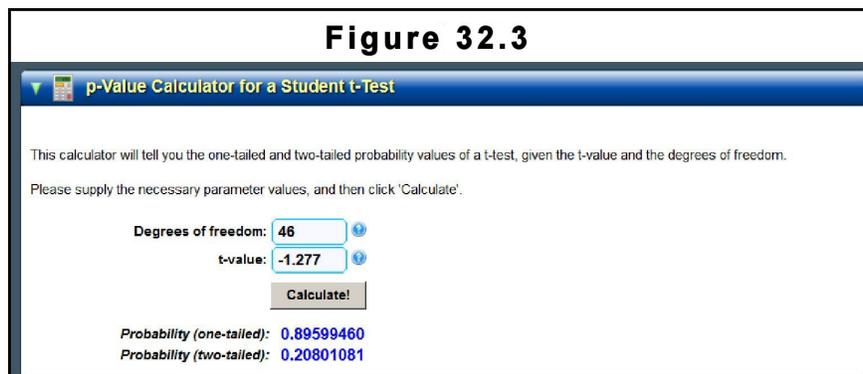
Z Test

Again review the methodology described in Chapter 23. If the standard deviation of the population is known, then one uses a z score rather than a t score. And,

according to some, but not to all, if the sample size is greater than 30, then one also uses a z score rather than a t score.

P Value

As an alternative to the traditional test, one may select an alpha level, say $\alpha=10\%$, and calculate the p-value. If $p>\alpha$ one fails to reject—that is, accepts— H_0 . For example, using a web-based calculator, for $DF=46$, the two-tail probability of a t score of -1.277 is 0.208 (See Figure 32.3). [1] And, $p=0.208$ is more than 10%. Thus, one does not reject H_0 .



Endnote

[1] Soper, D. (2015). *p-value calculator*. URL:<http://www.DanielSoper.com>

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