Type I Error

One of the purposes of inferential statistics is to compare a characteristic of a population —based on a random sample drawn from the population— to a hypothesized value in order to find out if the characteristic of the population and the hypothesized value are equal or are not equal. For example, one may compare the average height of the U.S. adult male population —based on a random sample drawn from the population — to a hypothesized value of 70 inches in order to find out if the the average height and the hypothesized value are equal or are not equal.

Simply stated, a type I error, which is known as a “false positive” error, occurs when one finds a difference when, in reality, there is none. Conversely, a type II error, which is known as a “false negative” error, occurs when one fails to find a difference when, in reality, there is one.

How does a type I error happen? This short paper describes a one-sample t test that exemplifies how a type I error may happen. The following sections describe the one-way t test.

One-Way T Test

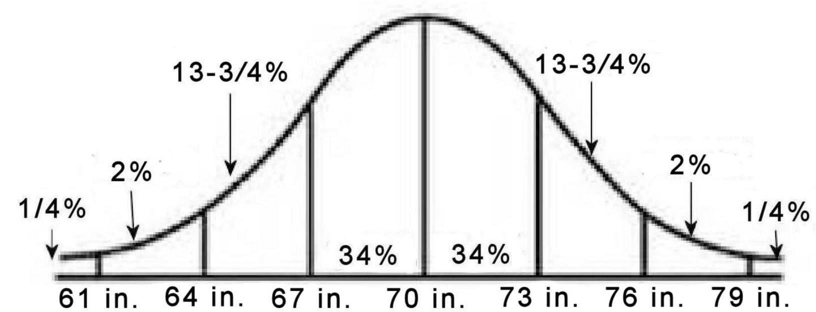
A t test used for comparing a hypothesized population mean to a sample mean, a statistic, which is a proxy for the population mean, a parameter, is known as a one-sample t test. The steps to be followed for a one-sample t test are:

- State the null hypothesis and the alternative hypothesis.
- Calculate the sample mean.
- Calculate the sample standard deviation.
- Calculate the relevant t score.
- Select a confidence level.
- Find the critical values that correspond to the selected confidence level.
- Fail to reject —that is, accept— or reject the null hypothesis.

Sample Data

In the real world, the average height of the U.S. adult male population is 70 inches and is normally distributed with a standard deviation of 3 (See Figure 1 on Page 2). For the purpose of this example, both the population's average height and standard deviation are unknown.

Figure 1



Most likely, a random sample of U.S. adult males would show that the average height is about 70 inches. However, although highly unlikely, is possible that the sample's average height is much lower than 70 inches or much higher than 70 inches. If so, based on the sample's average height, one may incorrectly infer that the population's average height is not 70 inches. This is an example of a type I error.

Table 1 shows a random sample of the height of 25 adult males in the United States, who, by chance, are taller than average.

Table 1				
Height (Inches)				
71.90	77.98	79.88	79.74	72.75
79.36	79.74	73.12	76.25	73.50
76.09	74.27	77.97	72.84	77.59
76.77	79.81	74.94	76.84	76.11
76.51	77.88	76.16	72.76	74.46

Null Hypothesis and Alternative Hypothesis

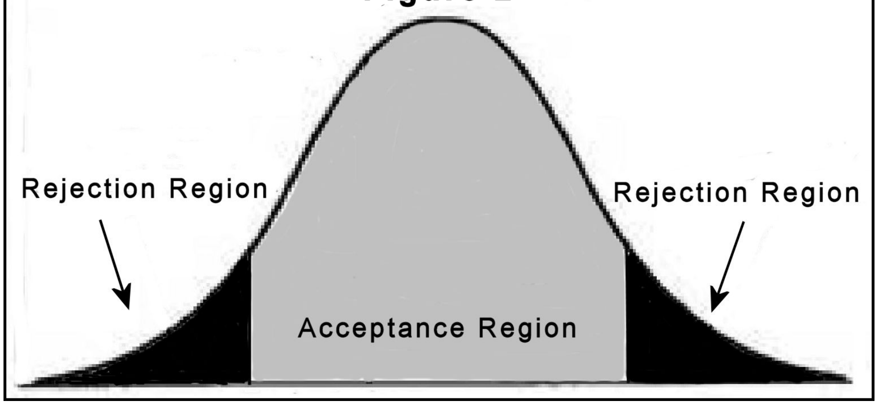
A hypothesis is a statement about a characteristic of a population —that is, the variable to be tested. Say that one states that the average height of the U.S. adult male population is equal to 70 inches. That is, H_0: $\mu=70$ and H_1: $\mu\neq70$. Is it true? Finding a difference between the stated value and the average height nullifies —that is, invalidates— the statement. Table 2 on Page 3 shows the null hypothesis and the alternative hypothesis.

Table 2		
	H_0	H_1
Bi-directional	μ=70”	μ≠70”

Hypothesis Testing

Figure 2 shows that, if the the test shows that the calculated t score is within the acceptance region, shown in gray, then, one fails to reject —that is, accepts— the null hypothesis and, therefore, is likely that there is no difference between the average height of the U.S. adult male population and 70 inches. Conversely, if the test shows that the the calculated t score is within the rejection regions, shown in black, then, one rejects the null hypothesis and, therefore, is likely that there is a difference between the average height and 70 inches.

Figure 2



Sample Mean

The sample mean of the distribution shown in Table 1 on Page 2 is 76.21:

$$(71.90+79.36+...+74.46)/25=76.21$$

Sample Standard Deviation

The sample standard deviation of the distribution shown in Table 1 on Page 2 is 2.52:

$$\mathrm{SQRT}(((71.90-70)^2+(79.36-70)^2+...+(74.46-70)^2)/24)=2.52$$

T Score

The formula for calculating the t score of a sample mean is …

$$t=\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

… where t is the t score, x-bar is the sample mean, μ is the population mean, which is the hypothesized mean, s is the standard deviation of the sample, and n is the sample size.

The relevant t score is 12.33:

(76.21-70)/(2.52/SQRT(25))=12.33

Degrees of Freedom

For this example, the degrees of freedom are 25-1=24.

Confidence Level and Critical Values

The t table is used for finding the critical values (CVs) of t that correspond to a selected level of confidence, where CL=100%-α, which, for this example, 97/5% is selected. Table 3 on Page 5 and Figure 3 on Page 5 show, for DF=24, that the t scores that correspond to the 97.5% confidence level, which is CL=100%-2.5%, are -2.39 for the left side and +2.39 for the right side. The confidence level shows the probability, which is, 97.5%, that the t scores are between -2.39 and +2.39. Note that α/2=-.0125%.

Table 3

Degrees of Freedom	Probability (One Tail)		
24	0.1000	0.0500	0.0250
	1.318	1.711	2.064
	Probability (Two Tail)		
	0.1000 (0.0500+0.0500)	0.0500 (0.0250+0.0250)	0.0250 (0.0125+0.0125)
	1.711	2.064	2.391

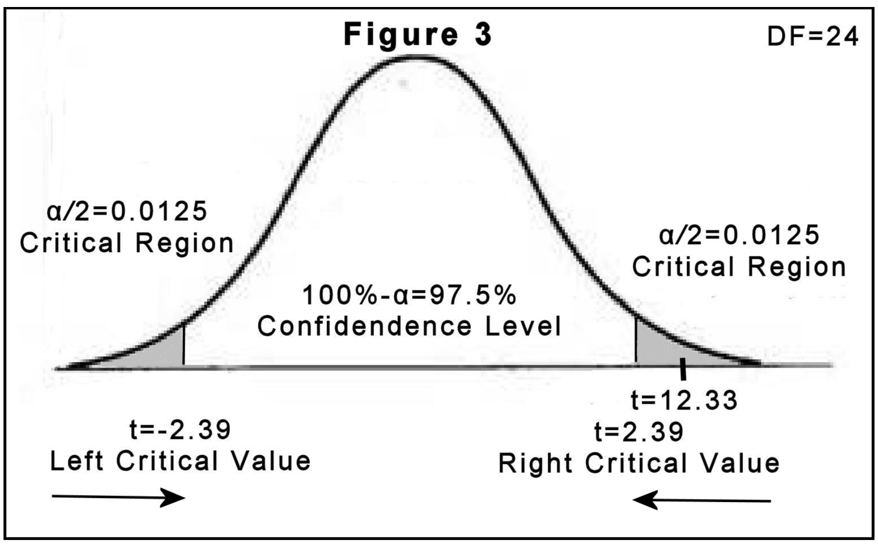


Figure 3

Inference

Figure 3 shows that, for DF=24, the t score of 12.33 is way outside the critical values of -2.39 and +2.39 and outside the 97.5% confidence level. Therefore, one incorrectly rejects the null hypothesis. This is an example of a type I error.

